EXPERIMENTAL METHOD OF DETERMINING PARAMETERS CHARACTERIZING THE MICROSTRUCTURE OF MICROPOLAR LIQUIDS

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A method is proposed for experimentally determining parameters characterizing the microstructure of a liquid. The method is based on measurement of the specific volumetric flow rates of the liquid as it is pumped through two capillary tubes of different cross section.

In theoretically investigating the hydrodynamics of liquids with an internal microstructure, especially wide use has been made of an approach based on the theory of micropolar liquids (MPL) [1]. However, there is very little information in the literature on values of the material constants which figure in the fundamental equations for the stress tensors and micromoments of the MPL. A method is proposed in [2] for determining coefficients characterizing the microstructure of liquids. Theoretical formulas were obtained for the coefficients $\varkappa$, μ, and γ, as well as the parameter $k = r_0[(2\mu+\varkappa)\varkappa/(\mu+\varkappa)\gamma]^{1/2}$. All of the quantities entering into these expressions are measured experimentally while pumping the test liquid through three flat capillary tubes with different cross sections. In deriving the formulas, the authors used full adhesion boundary conditions. Here, the translational velocity of the particles $\vec{v}$ and their microrotation on the surface $\vec{\nu}$ were equal, respectively, to the velocity and angular velocity of the boundary.

Following the same procedure, it is easy to obtain similar theoretical formulas with boundary conditions for $\vec{\nu}$ whereby the surface is free of moment stresses [3]. With more general boundary conditions [4, 5, 6], the formulas may include a quantity characterizing rotation of the particles on the wall. Since the question of the boundary conditions for the vector $\vec{\nu}$ has yet to be fully resolved, then the problem of experimentally determining the viscosity coefficients — or at least their complexes — is a very important one for description of the hydrodynamics of the steady-state flow of an MPL in capillary tubes.

We will examine a stabilized flow of an MPL under the influence of a constant pressure gradient dp/dz in a cylindrical capillary tube with an inside radius r_0. We ignore the compressibility of the liquid, as well as the body forces and moments. Let the physical properties of the MPL be constant. The system of differential equations for nontrivial components of the vectors $\vec{v}$ and $\vec{\nu}$ has the following form in the present case [1]:

$$(\mu+\varkappa)\frac{d}{dr}\left(r\frac{dv_z}{dr}\right)+\varkappa\frac{d}{dr}(r\nu_\varphi)=r\frac{dp}{dz}, \tag{1}$$

$$\gamma\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\nu_\varphi)\right]-\varkappa\frac{dv_z}{dr}-2\varkappa\nu_\varphi=0. \tag{2}$$

We use the following as the boundary conditions

$$\vec{v}(r_0)=0,\quad \vec{\nu}(r_0)=\frac{\alpha}{2}(\mathrm{rot}\,\vec{v})|_{r=r_0}, \tag{3}$$

where $0\le\alpha\le1$.

Solving system (1), (2) with boundary conditions (3), we obtain an expression for the velocity profile

$$v_z=\frac{r_0^2(-dp/dz)}{2(2\mu+\varkappa)}\left\{1-\tilde{r}^2+\delta\frac{I_0(k)}{kI_1(k)}\left[\frac{I_0(k\tilde{r})}{I_0(k)}-1\right]\right\}, \tag{4}$$

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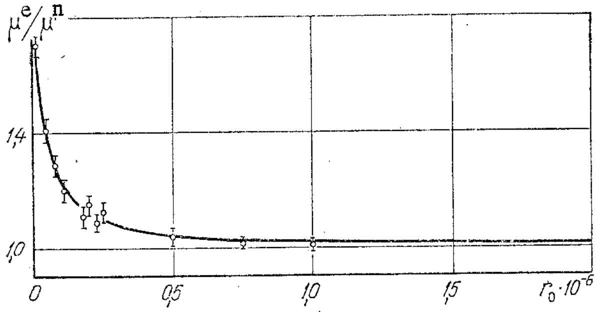

Fig. 1. Dependence of the ratio μ^e/μ^n for water on the inside radius of the capillary tube. $r_0 \cdot 10^{-6}$, m.

where $\tilde{r} = r/r_0$; $\delta = 2\varkappa(1-\alpha)/[2\mu + \varkappa(2-\alpha)]$; $I_0(k)$ and $I_1(k)$ are modified Bessel functions.

From (4) we find the specific volumetric flow rate of the micropolar liquid

$$Q = Q^n \left\{1 + \frac{4\delta}{k^2}\left[1 - \frac{kI_0(k)}{2I_1(k)}\right]\right\}. \tag{5}$$

Here $Q^n = r_0^2(-dp/dz)/8\mu^n$ is the flow rate for the flow of a Newtonian fluid with a shear viscosity $\mu^n = \mu + \varkappa/2$ in the same channel under the influence of a pressure gradient dp/dz. It should be noted that k increases with an increase in r_0 and that, in a sufficiently broad channel of radius $R_0 \gg r_0$, flow of the micropolar liquid is described by the Hagen–Poiseuille formula. Here, (5) changes into the expression

$$Q = \frac{R_0^2(-dp/dz)}{8\mu^n} = Q^n.$$

By measuring Q, R_0, and dp/dz in the present case, we can determine μ^n.

By pumping the investigated MPL with known pressure gradients $(dp/dz)_1$ and $(dp/dz)_2$ through two capillary tubes with internal radii r_0 and ar_0 $(0 < a < 1)$, we obtain

$$Q_1 = Q_1^n \left\{1 + \frac{4\delta}{k^2}\left[1 - \frac{kI_0(k)}{2I_1(k)}\right]\right\}, \quad Q_2 = Q_2^n \left\{1 + \frac{4\delta}{a^2k^2}\left[1 - \frac{akI_0(ak)}{2I_1(ak)}\right]\right\}.$$

From here

$$M\left[2I_1(ak) - akI_0(ak)\right] I_1(k) = \left[2I_1(k) - kI_0(k)\right] I_1(ak), \tag{6}$$

where

$$M = \frac{Q_2^n}{a^2 Q_1^n} \frac{Q_1 - Q_1^n}{Q_2 - Q_2^n}. \tag{7}$$

The parameter k, important in the hydrodynamics of the MPL, is determined from (6).

Analysis of the function

$$f(x) = \frac{2I_1(x) - xI_0(x)}{2I_1(ax) - axI_0(ax)} \frac{I_1(ax)}{I_1(x)}$$

shows that the range of its variation is narrower, the closer a is to unity. Since $f(k) = M$, we may make the following observations regarding the choice of the dimensions of the capillary tubes for the experiments. The values of M are obtained from experimental data which, of course, are associated with a certain error. This means that even a small error in the measurement of Q may lead to a large error in the calculation of k. Thus, it is necessary to choose capillary tubes with inside diameters corresponding to the lowest possible value of a.

From Eq. (5) we can determine the quantity

$$\delta = \frac{Q - Q^n}{2Q^n} \frac{k^2 I_1(k)}{2I_1(k) - kI_0(k)},$$

which, together with the values of k found earlier, gives us a complete quantitative description of the steady-state flow of the MPL in capillary tubes of any diameter. We should note the important fact that a specific value of a does not have to be assigned in order to calculate the parameters k and δ, which makes it possible in several cases to quantitatively describe the hydrodynamics of an MPL in channels with unknown boundary conditions.

Following the above procedure, it is easy to obtain an expression for determining the parameters k and δ in the case where the investigated liquid is pumped through two flat channels with distances of 2h and $2a$h between their walls. Here, the equation used to find k is as follows

$$M(1 - ak \operatorname{cth} ak) = 1 - k \operatorname{cth} k,$$

where $k = h[(2\mu+\varkappa)\varkappa/\gamma(\mu+\varkappa)]^{1/2}$, with M being determined from Eq. (7). The second parameter δ is calculated from the formula

$$\delta = \frac{Q - Q^{n}}{Q^{n}} \frac{2k^2}{3(1 - k \operatorname{cth} k)}.$$

The boundary conditions parameter, of course, characterizes the interaction of the particles of liquid both with the solid boundary and with each other. This is shown by the fact that it enters into δ in the form of a combination with $\varkappa$: $\delta = 2\varkappa(1 - \alpha)/[2\mu + \varkappa(1 - \alpha)]$. Thus, finding the value of the parameter α is just as important a problem as determining the viscosity coefficients of the MPL. Of possible promise in this regard for certain MPL's is the use of methods of creating "strong bonds" between the liquid particles and the boundary, similar to those described in [7] for nematic liquid crystals. Achieving full adhesion of the particles to the boundary would make it possible to determine both the coefficients $\varkappa$, μ, and γ and the quantity α for other boundary conditions.

At the same time, the possibility of experimentally determining the parameters $\tilde{k} = k/r_0$ and δ permits a quantitative description of the steady-state flow of the MPL in capillary tubes of any dimensions, regardless of the character of the boundary conditions.

As a specific example of using the proposed method of determining microstructural parameters, let us examine the results of [8]. Here, an experimental study was made of the phenomenon of an increase in the viscosity of polar liquids with a decrease in the radius of quartz capillary tubes. It was found that the flow of polar liquids in microcapillary tubes of internal radius $r_0 \leqslant 10^{-6}$ is not described by the Hagen–Poiseuille formula, and that the deviation from the formula is greater the smaller the radius. Thus, the volumetric flow rate of water through a capillary tube of radius $r_0 = 4 \cdot 10^{-8}$ m is 1.5 times lower than that calculated with the Hagen–Poiseuille formula when a constant tabulated value of the viscosity coefficient is inserted in the latter [8].

Proceeding on the basis of familiar representations of water as a structured liquid [9-12], we can attempt to explain these results by means of the MPL theory. In the flow of water in microcapillary tubes, polymolecular associations up to $3 \cdot 10^{-9}$ m in size which are present in the water [10] "twist" about one another. However, their interaction is accompanied by a continuous dissociation of old associations and creation of new ones. Thus, the theory of micropolar liquids can provide only a first approximation in describing the phenomenon of an increase in the viscosity of water with a decrease in capillary tube radius. Further studies are needed to determine the relationship which must exist between the lifetime of the associations and the time of their interaction for microrotations to be initiated. While not pretending that our examination of the phenomenon and its theoretical description by means of MPL theory is strictly adequate, we will illustrate the potential of the method proposed here for determining microstructural parameters and find them for water in a first approximation.

We will use the data from [8] (Fig. 1) on the flow of water in capillary tubes of internal radii $r_{01} = 3 \cdot 10^{-7}$ m and $r_{02} = a r_{01} = 5 \cdot 10^{-8}$ m $(a = 1/6)$. Since the form of Eq. (6) is fairly complex, to find the zero approximation of the solution we will use the following asymptotic representations of same:

$$k^2 = \frac{24(1 - Ma^2)}{2(Ma^4 - 1) + 3a^2(M - 1)} \quad (\text{small } k), \tag{8}$$

$$k = \frac{2(1 - M)}{1 - aM} \quad (\text{large } k). \tag{9}$$

The use of Eq. (9) and subsequent refinement of the resulting value with Eq. (6) leads to the following values of the sought parameters: $k = 21.2$, $\delta = 0.84$. The parameter $\tilde{k} = k/r_{o_1} = 70.3 \cdot 10^6\ m^{-1}$ is more convenient for practical use.

Figure 1 shows the curve, calculated with the above results, which depicts the capillary-tube-radius dependence of the ratio of the "equivalent" viscosity μ^e to the constant-volume viscosity of water, which coincides with μ^n. By equivalent we mean the viscosity of a Newtonian fluid flowing under the same conditions and at the same mean velocity as the investigated MPL. Comparison of the corresponding points of this curve with the experimental results in [8] shows a difference between them not exceeding 3%.

In the case where α is known, the values of $\varkappa$, μ, and γ can be determined. For example, given boundary conditions of full adhesion, with $\alpha = 0$, we have $\varkappa = 1.45 \cdot 10^{-3}\ kg \cdot m^{-1} \cdot sec^{-1}$, $\mu = 0.275 \cdot 10^{-3}\ kg \cdot m^{-1} \cdot sec^{-1}$, and $\gamma = 3.4 \cdot 10^{-19}\ kg \cdot m \cdot sec^{-1}$.

NOTATION

$\varkappa_\varphi$, μ, and γ, material constants characterizing momentum transfer in a micropolar liquid; α, boundary conditions parameter; v_z and $\varkappa_\varphi$, nontrivial components of the velocity and microrotation vectors, respectively; dp/dz, pressure gradient.

LITERATURE CITED

1. A. C. Eringen, "Theory of micropolar fluids," J. Math. Mech., 16, No. 1, 1-16 (1966).
2. N. P. Migun and V. L. Kolpashchikov, Vestn. Beloruss. State Univ., Ser. 1, No. 2, 83-85 (1977).
3. M. A. Turk, N. D. Sylvester, and T. Ariman, "On pulsative blood flow," Trans. Soc. Rheol., 17, No. 1, 1-21 (1973).
4. É. L. Aéro, A. N. Bulygin, and E. V. Kuvshinskii, "Asymmetrical hydromechanics," Prikl. Mat. Mekh., 29, No. 2, 297-308 (1965).
5. A. D. Kirwan and N. Newman, "Plane flow of a fluid containing rigid structures," Int. J. Eng. Sci., 7, No. 5, 883-893 (1969).
6. D. W. Condiff and J. S. Dahler, "Fluid mechanical aspects of antisymmetric stress," Phys. Fluids, 7, No. 6, 842-854 (1964).
7. P. G. De Gennes, Physics of Liquid Crystals, Oxford Univ. Press (1974).
8. B. V. Deryagin, B. V. Zheleznyi, Z. M. Zorin, V. D. Sobolev, and N. V. Churaev, "Properties of liquids in quartz capillaries," in: Surface Layers in Thin Films and the Stability of Colloids [in Russian], Nauka, Moscow (1974), pp. 90-94.
9. M. I. Shakhparonov, "Molecular interactions," in: Physics and Physical Chemistry of Fluids [in Russian], Moscow State Univ. (1976), pp. 35-43.
10. I. Z. Fisher and V. I. Adamovich, "Fluctuations of density in water," Zh. Strukt. Khim., 4, No. 6, 819-823 (1963).
11. M. S. John, J. Grosh, T. Ree, and H. Eyring, "Significant-structure theory applied to water and heavy water," J. Chem. Phys., 44, No. 4, 1465-1472 (1966).
12. Yu. V. Gurikov, "On the solubility of a mixture of nonpolar gases in water," Zh. Strukt. Khim., 10, No. 4, 583-588 (1969).